Lesson 62  Quadratic Equations with Complex Roots

Review: Shormann Algebra 2, Lessons 8, 34, 55; Shormann Algebra 1, Lesson 95

No New Rules or Definitions

From the fundamental theorem of algebra (Lesson 8), we know that a 2nd degree polynomial (quadratic polynomial) will have 2 complex roots. Until now, all the quadratic equations that you have solved have had 2 complex roots whose imaginary component equaled 0. In other words, they were real roots. Lesson 62 will really be more of the same, with the exception that, when you solve these quadratic equations, your roots will be true complex numbers (their imaginary component won’t equal zero). We talked briefly about Lesson 62 back in Lesson 34 when we introduced the discriminant. Finally, if you completed Shormann Algebra 1, this lesson is a review.

Example 62.1 Find the roots of \( x^2 + 4x + 7 = 0 \)

**solution:** We recognize this is a quadratic equation of the form \( ax^2 + bx + c = 0 \), so \( a = 1 \), \( b = 4 \), and \( c = 7 \). Apply the quadratic formula and solve:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(1)(7)}}{2} = \frac{-4 \pm \sqrt{-12}}{2} = \frac{-4 \pm \sqrt{-3} \times 2}{2} = \frac{-4 \pm 2\sqrt{3}i}{2} = -2 \pm \sqrt{3}i
\]

Since the roots of this equation are complex roots, that means if we graphed this as a function, \( f(x) = x^2 + 4x + 7 \), it would not cross the x-axis:

Another way to confirm this without graphing is by observing the discriminant (Lesson 34). Since the discriminant is a negative number (-12), that also indicates the equation will have two complex roots.

Finally, notice how the question asked you to “find the roots.” It didn’t ask you to find the zeros or x-intercepts. That’s because this equation doesn’t have places where it crosses the x-axis at \( y = 0 \). So, it wouldn’t make sense to say “find the zeros,” because this equation doesn’t have any! You’ll notice that the rest of the examples also ask you to find the roots, because that is the appropriate term to use now.
Example 62.2 Find the roots of $2x^2 + 3x + 6 = 0$

solution:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)6}}{2(2)} = \frac{-3 \pm \sqrt{-39}}{4} = \frac{-3 \pm \sqrt{39}i}{4}$$

Example 62.3 find the roots of $3x^2 - 4x + 9 = 0$

solution:

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)9}}{2(3)} = \frac{4 \pm \sqrt{-92}}{6} = \frac{4 \pm 4\sqrt{23}i}{6} = \frac{2 \pm \sqrt{23}i}{3}$$

Practice Set 62 (subscripts tell you which lesson each problem came from)

Use your best judgment as to when you should use a calculator. Use 3.14 for $\pi$.

1. Find the roots of $x^2 - 3x + 7 = 0$

2. How much water must be evaporated from 50 liters of a 10% salt solution to get a 40% salt solution?

3. Given: Circle $D$

   Tangents $BA$ and $BC$

   Prove: $\angle DAB \equiv \angle DCB$

   Use the SSS theorem in your proof.

4. Divide. $\frac{x^2 - 3}{x + 2}$

5. The kinetic energy, in Joules, of an object is described by the formula $K.E. = \frac{1}{2}mv^2$, where $m =$ mass in kg, and $v =$ the magnitude of the velocity in m/s. Find the velocity of a 1,000 kg object, if its kinetic energy equals $4.5 \times 10^6$ Joules. Round to 1 d.p.

6. The company's sales grew at an exponential rate, from $100 K$ initially to $900 K$ in the 4th year. Create an exponential equation, and then, assuming growth continues in an exponential pattern, use the equation to estimate the company's sales after 6 years. Round to the nearest $K$. 
7. Solve the following system.
\[
\begin{align*}
    x + y - 2z &= 16 \\
    2x - 3y + 7z &= 5 \\
    3x + 4y + 5z &= 93
\end{align*}
\]

8. Find the straight-line distance between (-3, -2) and (4, 7). Round to 1 d.p.

9. Use interval notation to describe \( x \geq 3 \).

10. Which of the following parabolic functions will have roots, but not zeros?
    
    A) \( f(x) = (x-2)^2 - 2 \)
    
    B) \( f(x) = -x^2 + 2 \)
    
    C) \( f(x) = (x + 2)^2 - 2 \)
    
    D) \( f(x) = (x - 2)^2 + 2 \)

11. Find the total surface area of the right solid shown. Dimensions are in cm. Round to the nearest whole number.

12. Solve.
\[
\frac{3}{4}x + \frac{1}{8} = \frac{1}{5} + \frac{x}{2}
\]

13. The horizontal lines are parallel, and cut by 2 transversals. Find \( x \). Round to 1 d.p.

14. The shadow of two buildings lands at the same location, S. Assuming the buildings are perpendicular to the ground, calculate the height of the taller building. The buildings are 112 ft apart, and the smaller building is 87 ft from S. Round to 1 d.p.
15. (SAT) If \( a = 3 \), what is the solution set of the equation shown? \( \sqrt{x - a} = x - 3 \)

Hint: Get rid of the radical sign first, then solve the resulting quadratic equation.
A) \{3\}  B) \{4\}  C) \{3, 4\}  D) \{-3, 0\}

16. To understand any subject well, not just math, one must start with rules and ____________________.

17. The triangles shown are similar. Find \( x \).

18. Find the roots of the polynomial equation shown. \( y = x^3 + 4x^2 - 21x \)

Assume it will factor into binomials.

19. Factor the quadratic expression into two binomials. \( x^2 + 16x + 63 \)

20. Solve for \( y \). \( 3ay + 8x = y - 2a \)